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A NONCLASSICAL MODEL FOR THE STRESSES IN 3-D CONTINUOUS FIBER-REINFORCED COMPOSITE MATERIALS

EMIN SELÇUK ARDIÇ, MICHAEL H. SANTARE and TSU-WEI CHOU
Department of Mechanical Engineering, University of Delaware, Newark, DE 19716, U.S.A.

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Abstract—A nonclassical continuum mechanics model is developed for a unidirectional continuous fiber-reinforced composite under elastic deformation to determine separate stress field expressions for each material in the body. Far-field strain components, measured or calculated by classical methods, are considered as known inputs. By an elasticity analysis the interactions between the adjacent material regions are found. By using a method similar to the nonlocal continuum mechanics theory all the long-range interactions are expressed as heterogeneity effects. By summing the stresses calculated from the far-field strains and the stresses caused by the heterogeneity effects, the stress fields in the different materials of the composite body are determined. Sample results are compared to those of a classical anisotropic elasticity method.

NOTATION

c_f	in-fiber heterogeneity effect variation factor
c_{ij}^m	in-matrix heterogeneity effect variation factor
E_{ij}	modulus of elasticity
e_{ij}	far-field strain components
e_{ij}'	nonlocal strains
f_{ij}	adjacent material region effects on the fibers
G_{ij}	shear modulus
m_{ij}	adjacent material region effects on the matrix
n	integer number of material regions
P	representative elastic constant for the mixture
Q_{ij}	mixture material moduli
R_2	inner characteristic length in x_2 direction
R_3	inner characteristic length in x_3 direction
R_f	fiber radius
R_{fi}	average dimensions for the fibers
R_{mi}	average dimensions for the matrix
R_m'	average matrix dimension for "23" component
S_{ij}^f	heterogeneity effects on the fibers
S_{ij}^m	heterogeneity effects on the matrix
s_{ij}^f	heterogeneity effect sign function for the fibers
s_{ij}^m	heterogeneity effect sign function for the matrix
δ_{ij}	Kronecker delta
ε_{ij}^f	average fiber strains
ε_{ij}^m	average matrix strains
η	correction factor for the modified rule-of-mixtures equations
λ_f	a fiber Lamé constant
λ_m	a matrix Lamé constant
μ_f	a fiber Lamé constant
μ_m	a matrix Lamé constant
Σ_{ij}^f	average heterogeneity effects on a fiber
Σ_{ij}^m	average heterogeneity effects on a matrix region
σ_{ijf}'	average heterogeneity stresses on the fibers
σ_{ijm}'	average heterogeneity stresses on the matrix
τ_{ij}	total stresses
τ_{ij}^0	local stresses
τ_{ijf}'	heterogeneity stresses on the fibers
τ_{ijm}'	heterogeneity stresses on the matrix.

1. INTRODUCTION

To analyze the stress fields in a composite body that consists of two or more different materials, several studies have been performed. Generally, these studies use a classical elasticity solution for a unit composite. Because of the complex interactions between the fibers and matrix, it is difficult to determine precisely the internal stress distribution for a whole body, especially a three-dimensional one. An extension of the unit composite solutions to several material regions is necessary to understand the mechanical properties of fiber-reinforced composites. In the following, material region is defined as one portion of the composite which consists entirely of fiber or matrix material.

Some previous studies analyze the internal stress distribution of materials reinforced with discontinuous fibers or inclusions or voids. These analyses are generally based on the shear-lag theory, classical elasticity theory and/or finite element analysis (i.e. Fukuda and Chou, 1981; Nemat-Nasser *et al.*, 1982). Others even though they are for continuous fiber-reinforced composites, use a unit composite approach (i.e. Amirbayat and Harle, 1969; Muki and Sternberg, 1970). Pagano (1969, 1970) found exact solutions for layered composites by expressing the displacements in the form of Fourier series. These two studies are limited to two-dimensional laminates and cannot be used to determine the stress distributions in the general three-dimensional case. In most of the studies concerning the stress or strain distributions in fiber-reinforced composites, the solutions for unit composites are extended to a whole body of fiber-reinforced composite by using a classical anisotropic elasticity analysis.

The classical solutions give average stresses for the whole body of a composite structure, but they cannot be used to distinguish the stress states in the different materials of the composite body. In other words, the classical methods consider the anisotropy but ignore the heterogeneity. The derivations and applications of the classical elasticity analysis for fiber-reinforced composites have been demonstrated, for example, by Vinson and Chou (1975), Vinson and Sierakowski (1987), and Tsai and Hahn (1980). Constitutive equations for incompressible, inextensible fiber-reinforced composites were derived by Pipkin and Rogers (1971), and this study was extended to three-dimensional case by Rogers (1974). These studies reduce to the classical anisotropic elasticity solutions for small deformations. Several other classical solutions, for example by Amirbayat and Hearle (1969), determine the stress fields for a unit composite but these solutions do not consider the interactions with the rest of the composite. In the present study, the strains which are calculated by the classical anisotropic elasticity, are considered to be the input far-field strains, therefore the anisotropy is regarded indirectly. By using a simple elasticity analysis for a unit composite and considering the effects caused by the heterogeneity, the separate stress fields for the fibers and matrix are determined and the solution is valid for the entire body. This technique was developd by Ardiç *et al.* (1989), for a two-dimensional laminated composite.

For a heterogeneous material a nonlocal continuum mechanics theory can be used to determine the field equations. Eringen (1977, 1987) summarized the theories of nonlocal continuum mechanics and applied them to a number of problems, and demonstrated the effectiveness of these theories by predicting various physical phenomena ranging from the global to the atomic scales. In the study presented in this paper, a method similar to the nonlocal theory and a simple elasticity analysis are used in a combined manner. Eringen (1977) states that an inner characteristic length should be associated with a given body and this length can be, for instance the average spacing distance between fibers in a composite body. In this study the average distances between the center lines of adjacent fiber and matrix regions in each direction are considered to be the inner characteristic lengths in those particular directions, and the constitutive equations which were derived by using the nonlocal continuum mechanics theory and presented, e.g. by Kröner (1967), and Eringen (1977, 1972, 1976) are used as a basis. By sampling the effects caused by the existence of the different materials in the body, at the center of each material region and using a classical elasticity analysis, all the material region interactions, which are called the heterogeneity effects in this study, are transferred to the strains and the local material moduli are used in the constitutive equations. Therefore, the method developed by Ardiç *et al.* (1989) is extended to the stress analysis of three-dimensional continuous fiber-reinforced composites. In this study, by using the heterogeneity effects inside each material region, and by satisfying the interface conditions and the average heterogeneity effects in the region of consideration, the stress variations inside the fiber or matrix regions are approximately determined. Since, there is no exact solution for a three-dimensional composite material, sample results of this study are compared to those from a classical anisotropic elasticity solution.

In Section 2 of this paper a simple elasticity analysis is presented, and the effects of adjacent material regions on each other are found. Heterogeneity effects are obtained in a microstructural scale and the stress/strain variations inside each material region are approximately determined, and the stresses caused by the heterogeneity are obtained in Section 3. In Section 4, by summing the local and heterogeneity stresses the resultant stress components for the different materials of the composite body are found. Section 5 offers a summary and conclusion of this study. In the Appendix the classical anisotropic elasticity solution is briefly explained.

2. EFFECTS OF ADJACENT MATERIAL REGIONS

A continuous fiber-reinforced composite material, assuming perfect bonding and uniform, periodic fiber distribution is considered. The stress fields in the fibers and matrix are to be found separately. The structure of the composite body is shown in Fig. 1.

It is assumed that the strain components for the composite as a whole are calculated by classical methods or measured, and therefore are known. The average stresses in each material region can be found by using a simple classical elasticity analysis and the matching conditions at the interface. In this problem the shaded matrix area and a quarter of the adjacent fiber in Fig. 1b are considered as a unit area. To find the strain components in each direction, a simplified geometry is used. An example for x_2 direction is shown in Fig. 2, where the relative areas are the same in both the original and simplified geometries. The average dimensions R_{f2}, R_{f3} and R_{m2}, R_{m3} can be found by using the following formulae:

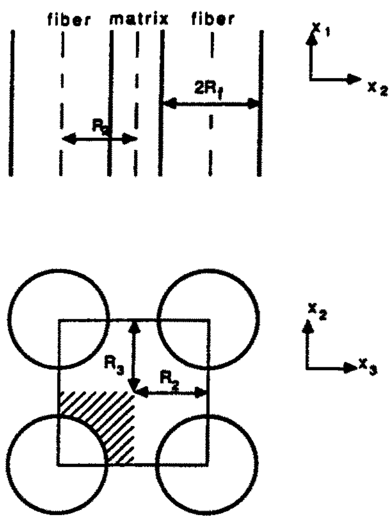

Fig. 1. Structure of the fiber-reinforced composite.

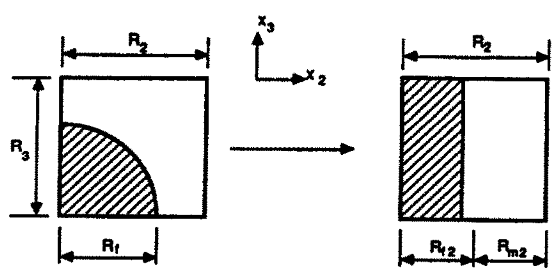

Fig. 2. Simplified geometry.

$$R_{f2} = \pi R_f^2/4R_3, \quad R_{f3} = \pi R_f^2/4R_2,$$
$$R_{m2} = R_2 - R_{f2}, \quad R_{m3} = R_3 - R_{f3}. \tag{1}$$

Here, the conditions over the unit material region are:

(i) the strain components along the fiber direction are the same for the fiber and matrix and they are equal to the corresponding input far-field strain component;
(ii) the averages of other strain components over the unit region in their particular direction are equal to the corresponding input far-field strain component;
(iii) tractions are equal at the interfaces, and it is assumed that the far-field strains are uniform inside each material region. Using these conditions, the average strain components for each material, except ε_{23} components, are found as follows:

$$\begin{aligned}
\varepsilon_{11}^f &= e_{11},\\
\varepsilon_{22}^f &= \{(\lambda_m - \lambda_f)R_{m2}e_{11} + (2\mu_m + \lambda_m)R_2 e_{22} + \lambda_m R_3 R_{m2} e_{33}/R_{m3}\\
&\quad - (\lambda_m R_{f3}/R_{m3} + \lambda_f)R_{m2}\varepsilon_{33}^f\}/\delta_2\\
\varepsilon_{33}^f &= (A_1 e_{11} + A_2 e_{22} + A_3 e_{33})\delta_0\\
\varepsilon_{31}^f &= \mu_m R_3 e_{31}/(\mu_m R_{f3} + \mu_f R_{m3})\\
\varepsilon_{12}^f &= \mu_m R_2 e_{12}/(\mu_m R_{f2} + \mu_f R_{m2})
\end{aligned} \tag{2}$$

for the matrix

$$\begin{aligned}
\varepsilon_{11}^m &= e_{11}\\
\varepsilon_{22}^m &= (R_2 e_{22} - R_{f2}\varepsilon_{22}^f)/R_{m2}\\
\varepsilon_{33}^m &= (R_3 e_{33} - R_{f3}\varepsilon_{33}^f)/R_{m3}\\
\varepsilon_{31}^m &= \mu_f R_3 e_{31}/(\mu_m R_{f3} + \mu_f R_{m3})\\
\varepsilon_{12}^m &= \mu_f R_2 e_{12}/(\mu_m R_{f2} + \mu_f R_{m2})
\end{aligned} \tag{3}$$

where e_{ij}'s are the far-field input strain components, and

$$\begin{aligned}
\delta_0 &= \delta_3 - (\lambda_m R_{f2} + \lambda_f R_{m2})(\lambda_m R_{f3} + \lambda_f R_{m3})/\delta_2\\
\delta_2 &= (2\mu_f + \lambda_f)R_{m2} + (2\mu_m + \lambda_m)R_{f2}\\
\delta_3 &= (2\mu_f + \lambda_f)R_{m3} + (2\mu_m + \lambda_m)R_{f3}\\
A_1 &= (\lambda_m - \lambda_f)R_{m2}R_{m3}[1 - (\lambda_m R_{f2} + \lambda_f R_{m2})/\delta_2]\\
A_2 &= R_2 R_{m3}[\lambda_m/R_{m2} - (\lambda_m R_{f2} + \lambda_f R_{m2})(2\mu_m + \lambda_m)/\delta_2]\\
A_3 &= R_3[(2\mu_m + \lambda_m) - (\lambda_m R_{f2} + \lambda_f R_{m2})\lambda_m/\delta_2].
\end{aligned} \tag{4}$$

To determine ε_{23} components for the fibers and matrix the averaging is done in both x_2 and x_3 directions. In this case the average dimension of a unit matrix region is found as

$$R'_m = (R_2 + R_3)/2 - R_f \tag{5}$$

therefore the ε_{23} components of the matrix and fiber strain tensors are:

$$\begin{aligned}
\varepsilon_{23}^f &= \mu_m(R_2 + R_3)e_{23}/2(\mu_m R_f + \mu_f R'_m)\\
\varepsilon_{23}^m &= \mu_f(R_2 + R_3)e_{23}/2(\mu_m R_f + \mu_f R'_m).
\end{aligned} \tag{6}$$

Using these strain expressions the effects of adjacent material regions on the material region of consideration are expressed as follows:

$$\begin{aligned} m_{11} &= 0, & m_{22} &= |\varepsilon^m_{22}-e_{22}|, & m_{33} &= |\varepsilon^m_{33}-e_{33}| \\ m_{23} &= |\varepsilon^m_{23}-e_{23}|, & m_{31} &= |\varepsilon^m_{31}-e_{31}|, & m_{12} &= |\varepsilon^m_{12}-e_{12}| \end{aligned} \tag{7}$$

$$\begin{aligned} f_{11} &= 0, & f_{22} &= |\varepsilon^f_{22}-e_{22}|, & f_{33} &= |\varepsilon^f_{33}-e_{33}| \\ f_{23} &= |\varepsilon^f_{23}-e_{23}|, & f_{31} &= |\varepsilon^f_{31}-e_{31}|, & f_{12} &= |\varepsilon^f_{12}-e_{12}|. \end{aligned} \tag{8}$$

The m_{ij}'s and f_{ij}'s represent the change in the corresponding strain components due to the existence of the adjacent material region.

3. HETEROGENEITY EFFECTS

In the nonlocal continuum mechanics literature, the stress field in an infinitesimal element is written as the sum of the local and nonlocal stresses as proposed by Eringen and Kröner, such that

$$\tau = \tau^0 + \tau' \tag{9}$$

where τ' is the nonlocal stress tensor and τ^0 is the local stress tensor which in this study is calculated from the known far-field strains, as if there is only one material (the material at the point of consideration). The general constitutive equation for the nonlocal stress field in a unidirectional continuous fiber-reinforced composite body, by following Eringen (1972, 1976, 1977, 1987) and Kröner (1967), can be written as

$$\tau'_{ij} = \iint (2\mu' e'_{ij} + \lambda' e'_{kk}\delta_{ij})\, \mathrm{d}x'_2\, \mathrm{d}x'_3 \tag{10}$$

where the fibers are in the x_1 direction, e'_{ij}'s are the components of the nonlocal strain tensor and x'_i represents the coordinates of all the points in the body except the point of consideration. μ' and λ' are the nonlocal material constants for a locally isotropic material as is considered in this study.

For the present case of unidirectional fiber-reinforced composite, each material region is affected by the adjacent material regions. The interactions, caused by the existence of different materials, occur as effects of effects in the directions perpendicular to the fibers. In this study these interactions are called the heterogeneity effects. By using the effects of the adjacent material regions, found in the previous section, all the heterogeneity effects are transferred to the strains and the local material moduli are used in the constitutive equations. Since the material region interactions are considered due to the heterogeneity, it is assumed that there is no heterogeneity effect in the fiber direction. For example, considering the e_{22} strain component for a stiff fiber adjacent to a compliant matrix region, the existence of matrix reduces that particular strain component in the fiber relative to the corresponding far-field strain. However, the next fiber has the effect of increasing the relative strain in the matrix, which in turn will increase the strain of the original fiber. Therefore, the second and higher order effects are taken into account. In this way the material region interactions can be expressed by the heterogeneity effects on strains. The direction dependence of the heterogeneity effects has already been imposed due to the orientation of the fibers, and only the distance dependence is considered. The effects of the equidistant fibers or matrix regions on either side of the region of consideration are written as a single term. For example, the following notation is used to represent the interaction effects on the "ij" component of the strain. For a stiff fiber and compliant matrix the term $m^n_{ij} f^{n-1}_{ij}$ represents the effect on the region of consideration (a matrix region for this sample case) due to the nth fibers. Similarly the term $-m_{ij}^n f_{ij}^n$ represents the effect from the nth matrix regions. Therefore, the heterogeneity effects can be summed in series as follows:

for the fibers

$$\begin{aligned}
\sigma'_{11f} &= \lambda_f(\Sigma_{22}^f + \Sigma_{33}^f) \\
\sigma'_{22f} &= (2\mu_f + \lambda_f)\Sigma_{22}^f + \lambda_f \Sigma_{33}^f \\
\sigma'_{33f} &= \lambda_f \Sigma_{22}^f + (2\mu_f + \lambda_f)\Sigma_{33}^f \\
\sigma'_{23f} &= 2\mu_f \Sigma_{23}^f \\
\sigma'_{31f} &= 2\mu_f \Sigma_{31}^f \\
\sigma'_{12f} &= 2\mu_f \Sigma_{12}^f
\end{aligned} \tag{11}$$

for the matrix

$$\begin{aligned}
\sigma'_{11m} &= \lambda_m(\Sigma_{22}^m + \Sigma_{33}^m) \\
\sigma'_{22m} &= (2\mu_m + \lambda_m)\Sigma_{22}^m + \lambda_m \Sigma_{33}^m \\
\sigma'_{33m} &= \lambda_m \Sigma_{22}^m + (2\mu_m + \lambda_m)\Sigma_{33}^m \\
\sigma'_{23m} &= 2\mu_m \Sigma_{23}^m \\
\sigma'_{31m} &= 2\mu_m \Sigma_{31}^m \\
\sigma'_{12m} &= 2\mu_m \Sigma_{12}^m
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
\Sigma_{ij}^f &= s_{\underline{ij}}^f \sum f_{\underline{ij}}^n (m_{\underline{ij}}^{n-1} - m_{\underline{ij}}^n) \\
\Sigma_{ij}^m &= s_{\underline{ij}}^m \sum m_{\underline{ij}}^n (f_{\underline{ij}}^{n-1} - f_{\underline{ij}}^n)
\end{aligned} \tag{13}$$

here the underbar means no summation. "n" represents the integer number of material regions. At this point the second and higher order heterogeneity terms in the summation series provide better accuracy. They can be used to determine the stress or strain variations inside each material region.

The heterogeneity stresses, expressed by eqns (11) and (12), are the average stresses, caused by the heterogeneity, in each material region. At this point the distribution of the heterogeneity effects inside each region is to be determined. Eringen (1977) stated that because of computational problems, manageable nonlocal moduli distribution functions should be selected. In the problem presented in this paper, at this point the average heterogeneity effects in a particular region and the heterogeneity effects at the interfaces, not nonlocal moduli, are known. It is also known that the stress distribution in a material region will be in the form of a continuous and smooth function. For simplicity, in this study a square root function of the distance from the interface is selected. The geometry of the system for two arbitrary points is shown in Fig. 3, the stresses are assumed to be found at these points. Therefore, the heterogeneity stresses can be written as follows:

for the fibers

$$\begin{aligned}
\tau'_{11f} &= \lambda_f(S_{22}^f + S_{33}^f) \\
\tau'_{22f} &= (2\mu_f + \lambda_f)S_{22}^f + \lambda_f S_{33}^f \\
\tau'_{33f} &= \lambda_f S_{22}^f + (2\mu_f + \lambda_f)S_{33}^f \\
\tau'_{23f} &= 2\mu_f S_{23}^f
\end{aligned}$$

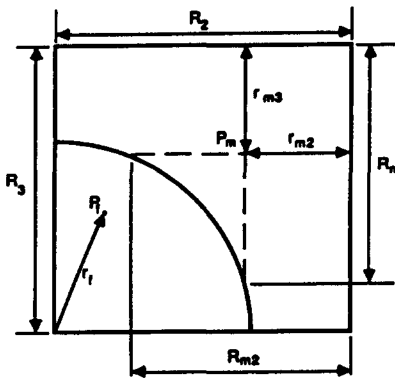

Fig. 3. Inner dimensions for two arbitrary points, P_f and P_m, in the fibers and matrix.

$$\begin{aligned}
\tau'_{31f} &= 2\mu_f S^f_{31}\\
\tau'_{12f} &= 2\mu_f S^f_{12}
\end{aligned}\tag{14}$$

and for the matrix

$$\begin{aligned}
\tau'_{11m} &= \lambda_m(S^m_{22}+S^m_{33})\\
\tau'_{22m} &= (2\mu_m+\lambda_m)S^m_{22}+\lambda_m S^m_{33}\\
\tau'_{33m} &= \lambda_m S^m_{22}+(2\mu_m+\lambda_m)S^m_{33}\\
\tau'_{23m} &= 2\mu_m S^m_{23}\\
\tau'_{31m} &= 2\mu_m S^m_{31}\\
\tau'_{12m} &= 2\mu_m S^m_{12}
\end{aligned}\tag{15}$$

where

$$\begin{aligned}
S^f_{ij} &= s^f_{\underline{ij}}\{f_{\underline{ij}}(1-c_f m_{\underline{ij}})+c_f\textstyle\sum f^n_{\underline{ij}}(m^{n-1}_{\underline{ij}}-m^n_{\underline{ij}})\}\\
S^m_{ij} &= s^m_{\underline{ij}}\{m_{\underline{ij}}(1-c^m_{\underline{ij}}f_{\underline{ij}})+c^m_{\underline{ij}}\textstyle\sum m^n_{\underline{ij}}(f^{n-1}_{\underline{ij}}-f^n_{\underline{ij}})\}
\end{aligned}\tag{16}$$

here, "n" again represents the number of fiber and matrix regions away from the material region of consideration, the underbar means no summation over the indices, and

$$\begin{aligned}
c_f &= 15[(R_f-r_f)/R_f]^{1/2}/8\\
c^m_{22} &= 3[(R_{m2}-r_{m2})/R_{m2}]^{1/2}/2\\
c^m_{33} &= 3[(R_{m3}-r_{m3})/R_{m3}]^{1/2}/2\\
c^m_{23} &= 3(R_2+R_3-R_f)[(R_{m2}-r_{m2})^{1/2}+(R_{m3}-r_{m3})^{1/2}]/[2(R^{3/2}_{m2}+R^{3/2}_{m3})]\\
c^m_{31} &= 3[(R_{m3}-r_{m3})/R_{m3}]^{1/2}/2\\
c^m_{12} &= 3[(R_{m2}-r_{m2})/R_{m2}]^{1/2}/2.
\end{aligned}\tag{17}$$

c^m_{ij}'s and c_f represent the variation of the heterogeneity effects inside each material region. They are determined by using the conditions that the tractions will be equal at the interface and the average heterogeneity effects will equal those given in eqns (13). Because of the symmetric geometry of the fiber cross section c_f is the same for all components.

Therefore, the stress components, caused by the heterogeneity effects, are determined, now the resultant total stresses will be found.

4. TOTAL STRESSES AND RESULTS

The heterogeneity stresses for the fibers and matrix were found and expressed by eqns (14) and (15). The local stress tensor is calculated from the known far-field strain components, as if there is only one material (the material at the point of consideration). The local stress field equations can be written as

$$\begin{aligned} \tau^0_{ijf} &= 2\mu_f e_{ij} + \lambda_f \delta_{ij} e_{kk} \\ \tau^0_{ijm} &= 2\mu_m e_{ij} + \lambda_m \delta_{ij} e_{kk}. \end{aligned} \tag{18}$$

The resultant stress components are the sum of the local and the heterogeneity stress components. Therefore, the total stress field expressions are obtained as follows:

for the fibers

$$\begin{aligned} \tau^f_{11} &= (2\mu_f+\lambda_f)e_{11}+\lambda_f(e_{22}+e_{33})+\lambda_f(S^f_{22}+S^f_{33}) \\ \tau^f_{22} &= (2\mu_f+\lambda_f)e_{22}+\lambda_f(e_{11}+e_{33})+(2\mu_f+\lambda_f)S^f_{22}+\lambda_f S^f_{33} \\ \tau^f_{33} &= (2\mu_f+\lambda_f)e_{33}+\lambda_f(e_{11}+e_{22})+\lambda_f S^f_{22}+(2\mu_f+\lambda_f)S^f_{33} \\ \tau^f_{23} &= 2\mu_f e_{23}+2\mu_f S^f_{23} \\ \tau^f_{31} &= 2\mu_f e_{31}+2\mu_f S^f_{31} \\ \tau^f_{12} &= 2\mu_f e_{12}+2\mu_f S^f_{12} \end{aligned} \tag{19}$$

for the matrix

$$\begin{aligned} \tau^m_{11} &= (2\mu_m+\lambda_m)e_{11}+\lambda_m(e_{22}+e_{33})+\lambda_m(S^m_{22}+S^m_{33}) \\ \tau^m_{22} &= (2\mu_m+\lambda_m)e_{22}+\lambda_m(e_{11}+e_{33})+(2\mu_m+\lambda_m)S^m_{22}+\lambda_m S^m_{33} \\ \tau^m_{33} &= (2\mu_m+\lambda_m)e_{33}+\lambda_m(e_{11}+e_{22})+\lambda_m S^m_{22}+(2\mu_m+\lambda_m)S^m_{33} \\ \tau^m_{23} &= 2\mu_m e_{23}+2\mu_m S^m_{23} \\ \tau^m_{31} &= 2\mu_m e_{31}+2\mu_m S^m_{31} \\ \tau^m_{12} &= 2\mu_m e_{12}+2\mu_m S^m_{12} \end{aligned} \tag{20}$$

where the heterogeneity effects, S_{ij}'s, are given in eqns (16) and (17).

Numerical results are obtained by using the eqns (19) and (20). These results are compared to those of a classical anisotropic elasticity solution for some sample far-field strain components. The nonclassical solution can be used for any kind of far-field strains, but in this study for the sake of simplicity only linear variations in x_2-direction are considered. For a given strain component, the variations in stress components with distance for a classical anisotropic elasticity solution and the nonclassical approach, developed in this study, are shown in Figs. 4, 5, 6, 7. The materials for this sample problem are graphite fibers ($E_f = 275.86$ GPa, $\nu_f = 0.2$) and polyimide matrix ($E_m = 2.76$ GPa, $\nu_m = 0.33$). The fiber volume fraction is 50%. In Figs 4 through 7 the stress components are considered in the planes passing through the center lines of the fibers, thus in these figures, "$1.0 \leqslant x_2/R_2$ (or x_3/R_3) $\leqslant 2.5958$" and "$3.0 \leqslant x_2/R_2$ (or x_3/R_3) $\leqslant 4.5958$" are graphite fiber regions, and "$2.5958 \leqslant x_2/R_2$ (or x_3/R_3) $\leqslant 3.0$" and "$4.5958 \leqslant x_2/R_2$ (or x_3/R_3) $\leqslant 5.0$" are polyimide matrix regions where R_2 and R_3 are as shown in Figs 1 and 2. The classical anisotropic elasticity solution results are obtained by using the anisotropic constitutive equations presented in Chapter 2 of Vinson and Sierakowski (1987). To determine the reduced elastic constants for the classical solution, the modified rule-of-mixtures equations which are given by Tsai and Hahn (1980) and Vinson and Sierakowski (1987), are used. A brief summary of the classical solution is presented in the Appendix of this paper.

In Figs 4 through 7, it is seen that the classical anisotropic elasticity solution gives smooth, continuous curves. Since this classical solution considers the anisotropy and ignores

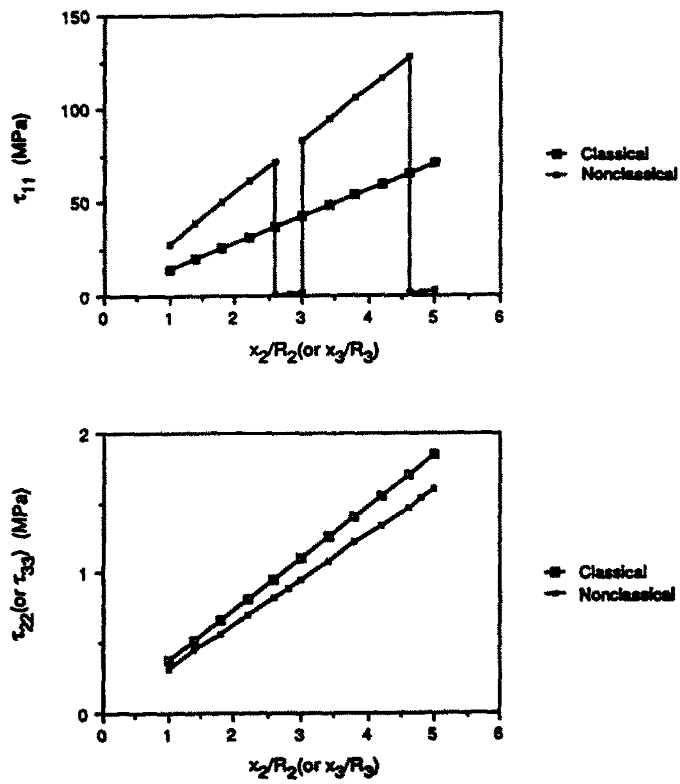

Fig. 4. Stress components vs distance for the far-field strain components, $e_{11} = 0.0001(x_2/R_2)$, other $e_{ij} = 0$.

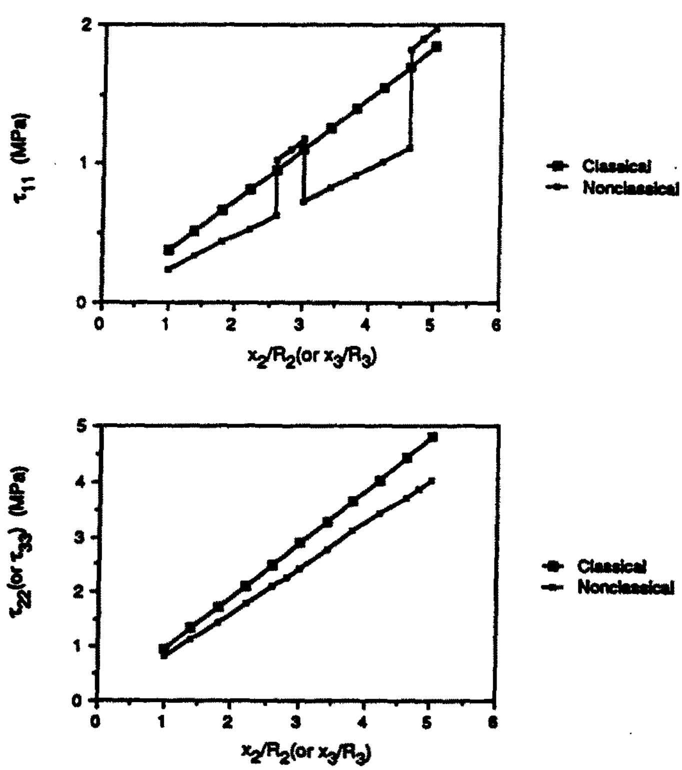

Fig. 5. Stress components vs distance for the far-field strain components, $e_{22} = 0.0001(x_2/R_2)$, other $e_{ij} = 0$ (or $e_{33} = 0.0001(x_3/R_3)$, other $e_{ij} = 0$).

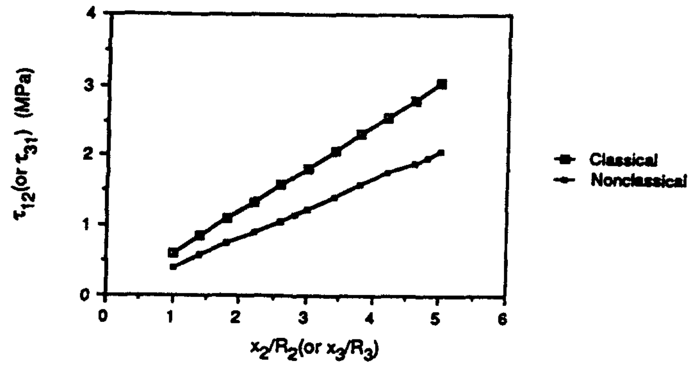

Fig. 6. Stress components vs distance for the far-field strain components, $e_{12} = 0.0001(x_2/R_2)$, other $e_{ij} = 0$ (or $e_{31} = 0.0001(x_3/R_3)$, other $e_{ij} = 0$).

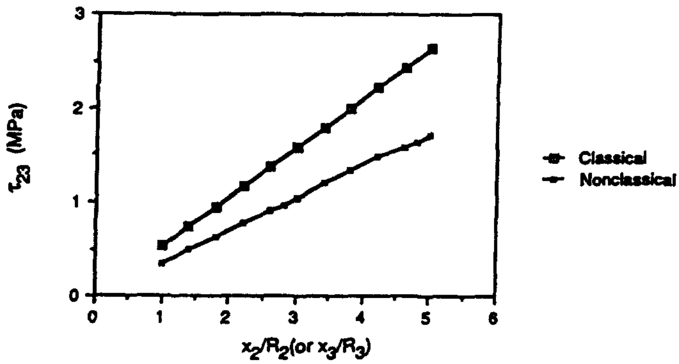

Fig. 7. Stress components vs distance for the far-field strain components, $e_{23} = 0.0001(x_2/R_2)$ or $e_{23} = 0.0001(x_3/R_3)$, other $e_{ij} = 0$.

the heterogeneity it cannot distinguish the stress states for the different materials of the composite body. The nonclassical approach gives continuous curves except for the stress components in the fiber direction, and the variations inside each material region are determined approximately by using the heterogeneity effects approach. It gives discontinuous curves for the stress components in the fiber direction because in this approach the stress values are distinguished for the different material regions in the composite. The nonclassical solution considers the heterogeneity and takes the anisotropy into account by the use of the input far-field strains.

5. CONCLUSION

In this study a nonclassical formulation is used with a simple classical elasticity solution in a combined manner to determine separate stress fields for the fibers and matrix in a continuous fiber-reinforced composite material. Sample results were presented for some special input far-field strains and the results were compared to those of a classical anisotropic elasticity solution. The nonclassical solution method presented in this paper can be used to determine separate stress fields for the fibers and matrix and also the stress variations inside each material region can be found approximately. By using the nonclassical solution method both heterogeneity and anisotropy in the body of composite material are taken into account, thus this solution is valid for the entire body of composite material. Therefore, the nonclassical method can have practical application in some realistic problems. In a previous study by Ardiç *et al.* (1989) the accuracy of the method was shown for two-dimensional problems. As yet, the accuracy of the three-dimensional solution is unverified.

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APPENDIX

Classical anisotropic elasticity solution

First, the reduced elastic constants are determined by using the equations which are called modified rule-of-mixture equations by Tsai and Hahn (1980), as given by Vinson and Sierakowski (1987). To determine the material constants of the composite the general expression

$$P = (P_f V_f + \eta P_m V_m)/(V_f + \eta V_m)$$

is used. In this expression P represents the elastic constants of the mixture, P_f and P_m are the corresponding material constants of the fibers and matrix, respectively. V_f and V_m are the volume fractions (whose sum equals unity). The forms of P, P_f, P_m and η are tabulated in Chapter 2 of Vinson and Sierakowski (1987).

The general constitutive equations for a lamina of fiber-reinforced composite material or for laminates which are all reinforced in the same direction, by ignoring the effects of the interfaces between laminates, can be written as given by Vinson and Sierakowski (1987), such as

$$\begin{aligned}
\sigma_{11} &= Q_{11}e_{11} + Q_{12}e_{22} + Q_{13}e_{33} \\
\sigma_{22} &= Q_{12}e_{11} + Q_{22}e_{22} + Q_{23}e_{33} \\
\sigma_{33} &= Q_{13}e_{11} + Q_{23}e_{22} + Q_{33}e_{33} \\
\sigma_{23} &= 2Q_{44}e_{23} \\
\sigma_{31} &= 2Q_{55}e_{31} \\
\sigma_{12} &= 2Q_{66}e_{12}
\end{aligned}$$

where

$$\begin{aligned}
Q_{11} &= E_{11}(1-\nu_{23}\nu_{32})/\Delta \\
Q_{22} &= E_{22}(1-\nu_{13}\nu_{31})/\Delta \\
Q_{33} &= E_{33}(1-\nu_{12}\nu_{21})/\Delta \\
Q_{44} &= G_{23}, \quad Q_{55} = G_{13}, \quad Q_{66} = G_{12} \\
Q_{12} &= (\nu_{21}+\nu_{31}\nu_{23})E_{11}/\Delta = (\nu_{12}+\nu_{32}\nu_{13})E_{22}/\Delta \\
Q_{13} &= (\nu_{31}+\nu_{21}\nu_{32})E_{11}/\Delta = (\nu_{13}+\nu_{12}\nu_{23})E_{33}/\Delta \\
Q_{23} &= (\nu_{32}+\nu_{12}\nu_{31})E_{22}/\Delta = (\nu_{23}+\nu_{21}\nu_{13})E_{33}/\Delta \\
\Delta &= 1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-2\nu_{21}\nu_{32}\nu_{13}.
\end{aligned}$$